Probability and Random Processes

ECS 315

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1 Probability and You

Office Hours:

BKD, 6th floor of Sirindhralai building

Wednesday 13:45-15:15

Friday 13:45-15:15

Everything we do, everything that happens around us, obeys the laws of **probability**.

We can no more escape them than we can escape gravity… "Probability," a philosopher (Bishop Butler) once said, "is the **very guide of life**."

We are all gamblers who go through life making countless bets on the outcome of countless actions.

[Gardner, 1986]

Life is random

In 2005, this statement (which is true)

Life is random

was on display all over the world…

Life is random

[https://www.youtube.com/watch?v=0d8aVzQY4Zo]

iPod shuffle

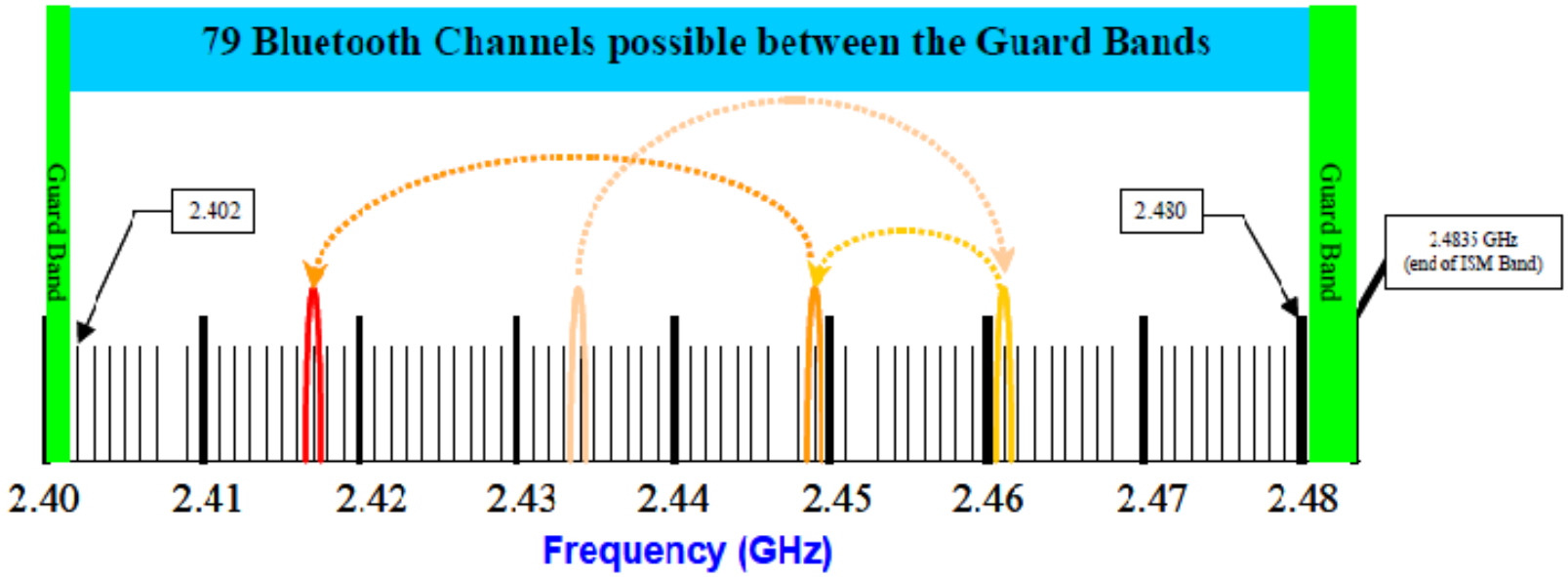
Life is random.

Applications of Probability Theory

- The subject of probability can be traced back to the 17th century when it arose out of the study of **gambling games**.
- The range of applications extends beyond games into business decisions, insurance, law, **medical tests**, and the social sciences.
- The **stock market**, "the largest casino in the world," cannot do without it.
- The **telephone network**, call centers, and airline companies with their randomly fluctuating loads could not have been economically designed without probability theory.

FHSS Example: Bluetooth

- The band at 2.4 GHz is divided into 79 channels.
- A Bluetooth device, hops frequency at a rate of 1600 hops per second, randomly selecting a channel of 1 MHz to operate.



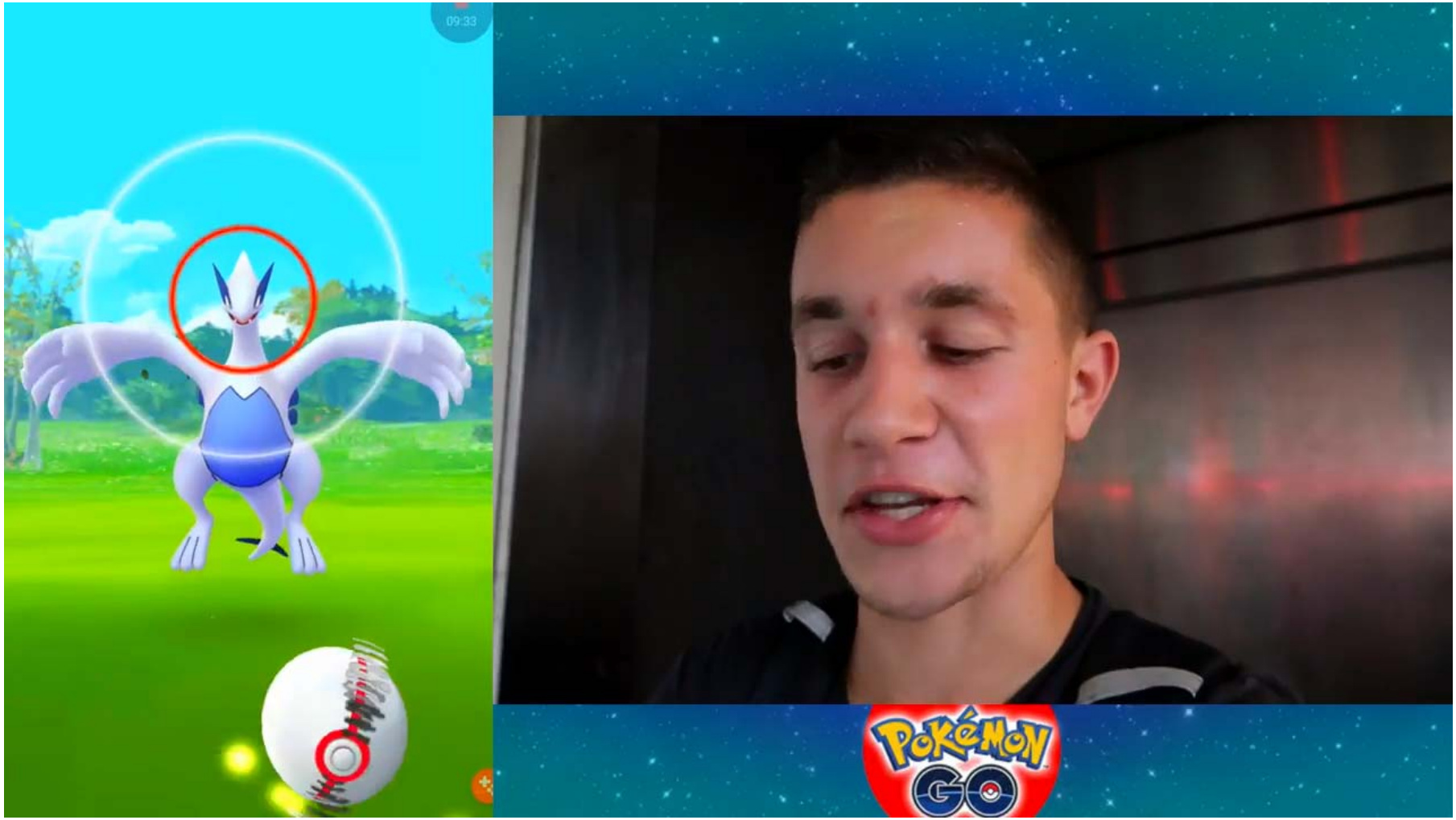
[https://sites.google.com/site/nearcommunications/adaptative-frequency-hopping]

Pokémon GO

- Catch rate
- Flee rate

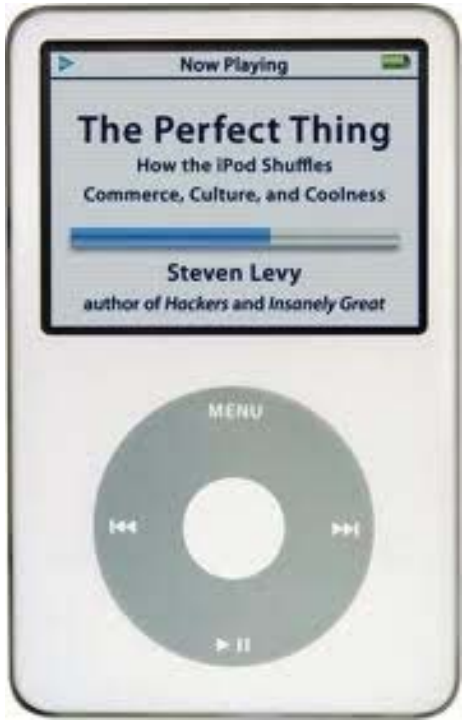
[https://www.youtube.com/watch?v=6VOl8Hm4tG0]

Pokémon GO



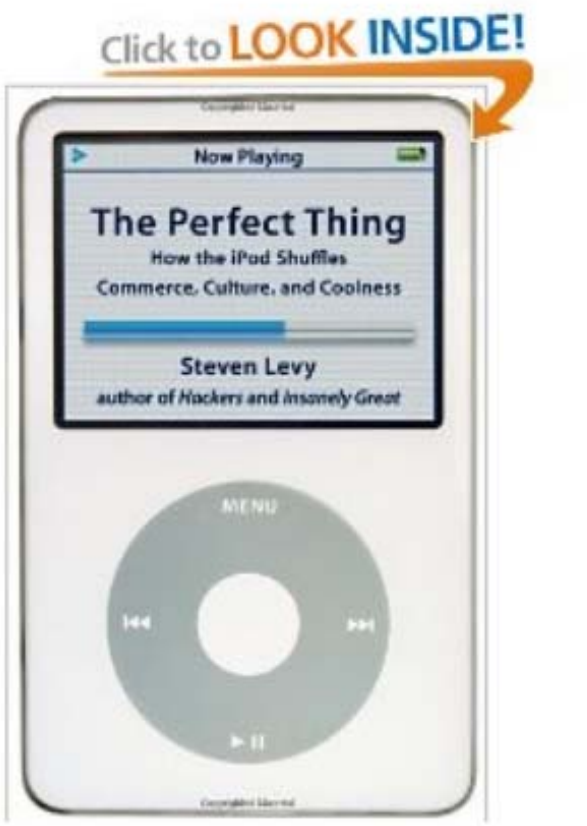
[https://www.youtube.com/watch?v=6VOl8Hm4tG0]

“The Perfect Thing”



What is this?

“The Perfect Thing”



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Steven Levy (Author)

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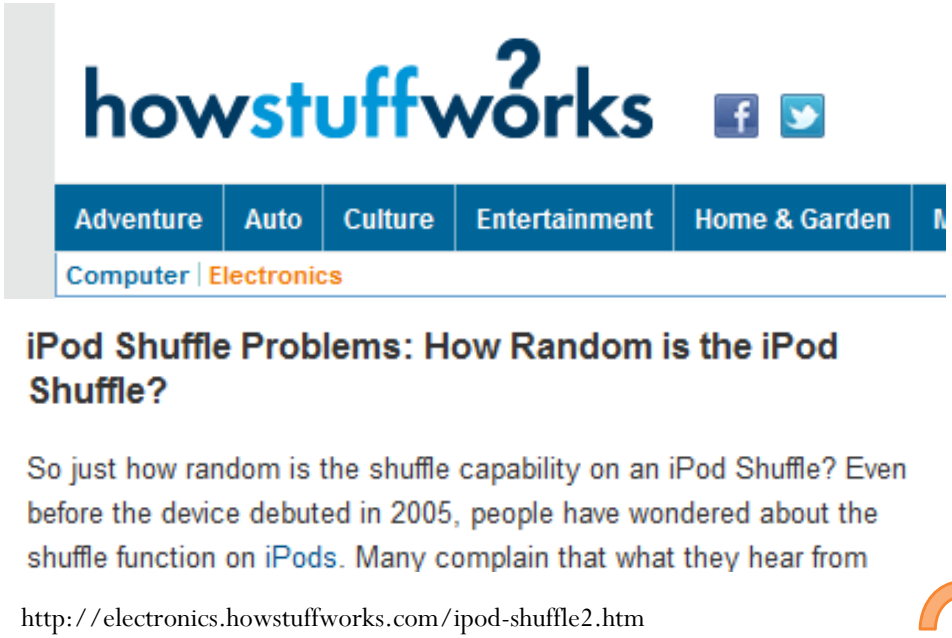
What about the shuffle function?

http://ipod.about.com/od/advanceditunesuse/a/itunes-random.htm

http://www.cnet.com.au/itunes-just-how-random-is-random-339274094.htm



http://electronics.howstuffworks.com/ipod-shuffle2.htm

USA Currency Coins

- Penny = 1 cent
  (Abraham Lincoln)

- Nickel = 5 cents
  (Thomas Jefferson)

- Dime = 10 cents
  (Franklin D. Roosevelt)

- Quarter = 25 cents
  (George Washington)

Thai Coins

Front

Back

[http://travel.kapook.com/view75126.html]

`randi` function

- Generate uniformly distributed **pseudorandom integers**
- `randi(imax)` returns a scalar value between 1 and `imax`.
- `randi(imax,m,n)` and `randi(imax,[m,n])` return an *m*-by-*n* matrix containing pseudorandom integer values drawn from the discrete uniform distribution on the interval [1,`imax`].
  - `randi(imax)` is the same as `randi(imax,1)`.
- `randi([imin,imax],...)` returns an array containing integer values drawn from the discrete uniform distribution on the interval [`imin`,`imax`].

`randi` function: examples

Coin Tosses:

```
>> randi([0,1])
ans =        T, H
     0
>> randi([0,1],10,2)
ans =
     1     0
     1     0
     1     0
     1     1
     1     1
     0     0
     1     1
     0     0
     1     0
     0     0
```

Dice Rolls

```
>> randi([1,6])
ans =
     5
>> randi([1,6],10,2)
ans =
     5     1
     2     1
     3     3
     3     6
     4     3
     5     4
     5     2
     2     5
     5     2
     4     4
```


`randi` function: examples

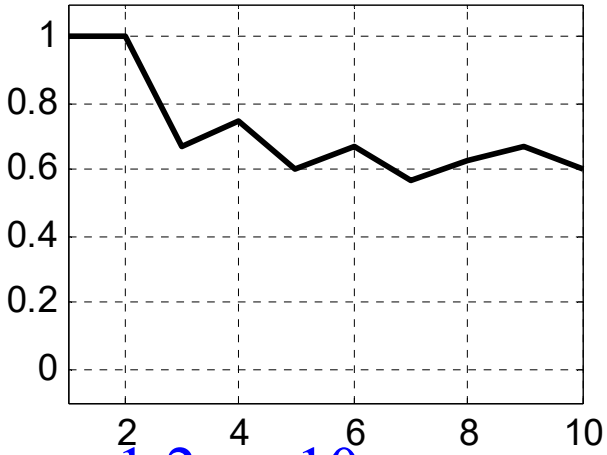
Coin Tosses:

```
>> S = ['T','H']
S =
TH
>> S(randi([1,2]))
ans =
H
>> S(randi([1,2],10,2))
ans =
TT
HH
HT
TT
HT
TT
TH
HT
HH
HT
```

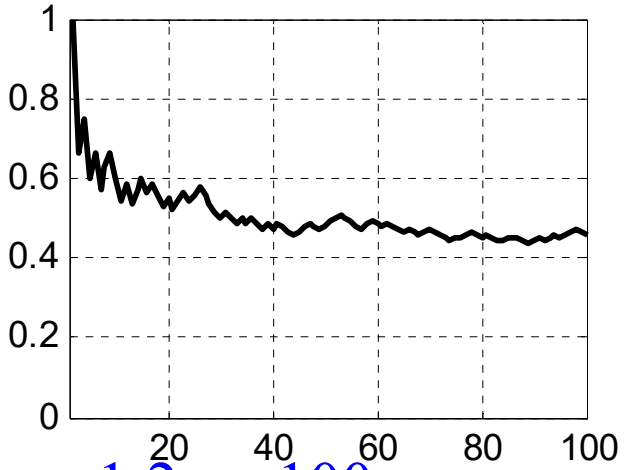

Coin Tossing: Relative Frequency

$$\frac{N(A,n)}{n}$$

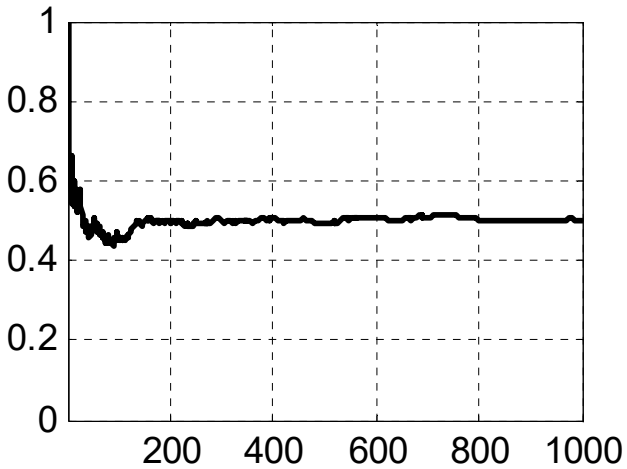
If a fair coin is flipped a large number of times, the **proportion** of heads will tend to get closer to 1/2 as the number of tosses increases.



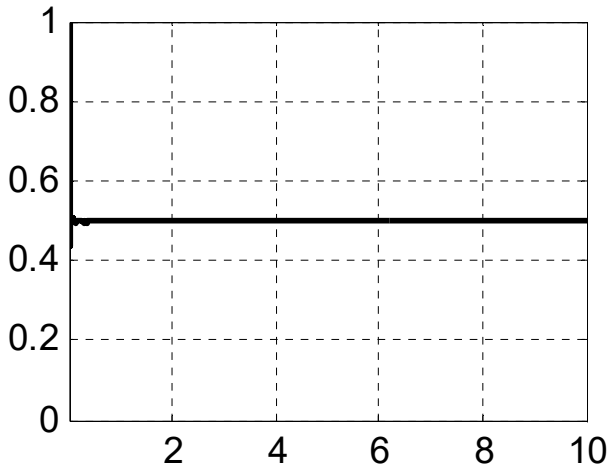
$n = 1, 2, \ldots, 10$



$n = 1, 2, \ldots, 100$



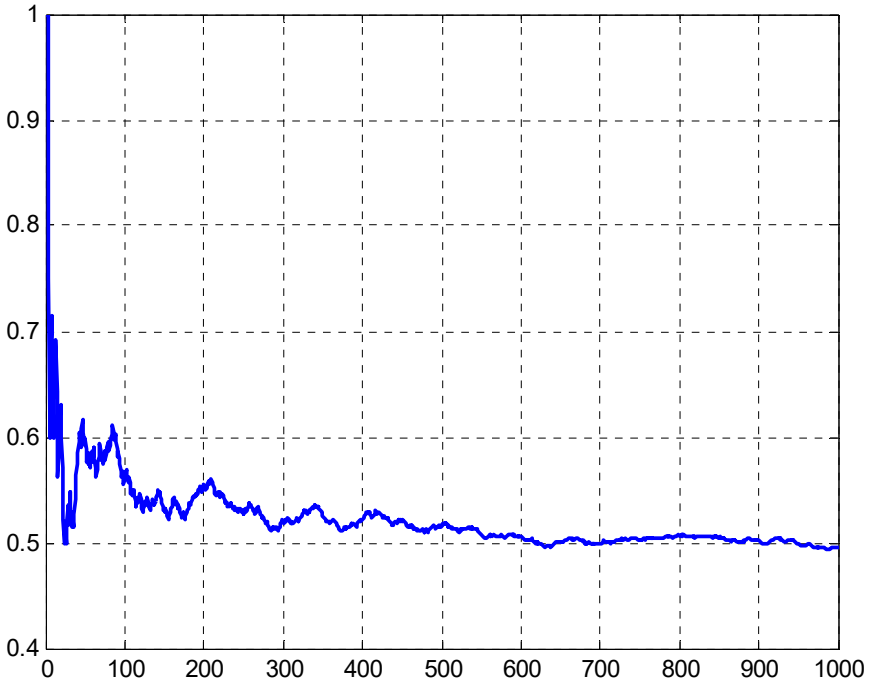
$n = 1, 2, \ldots, 1000$



$n = 1, 2, \ldots, 10^6$

Coin Tossing: Relative Frequency

```
close all; clear all;
N = 1e3; % Number of trials (number of times that the coin is tossed)
s = randi([0,1],1,N);   % Generate a sequence of N Coin Tosses.
                        % The results are saved in a row vector s.
NH = cumsum(s); % Count the number of heads
plot(NH./(1:N),'LineWidth',1.5); grid on % Plot the relative frequencies
```



Coin Tossing: Relative Freq. vs. #H-#T

This statement does not say that the difference between #H and #T will be close to 0.

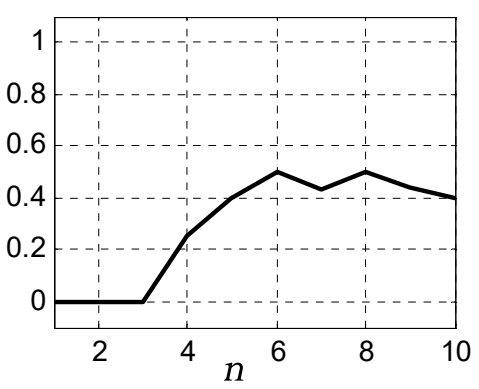
If a fair coin is flipped a large number of times, the **proportion** of heads will tend to get closer to 1/2 as the number of tosses increases.

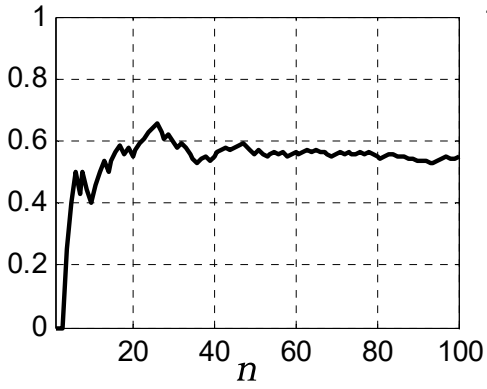
The **difference** between #H and #T will **not** converge to 0.

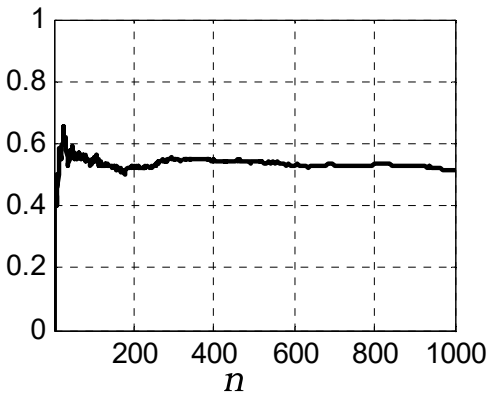
Another Experiment

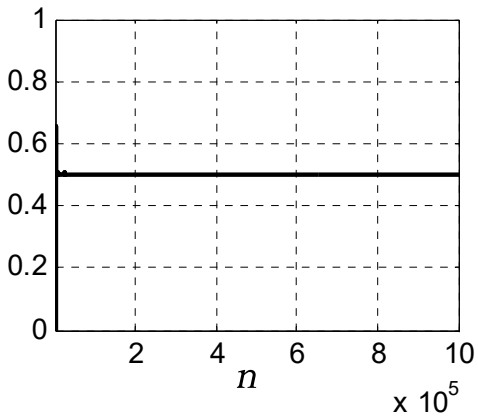
Relative Freq.

#H-#T









Another Experiment

Relative Freq.

#H-#T